

# Markscheme

**May 2023**

**Physics**

**Higher level**

**Paper 3**

26 pages

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### Subject Details: Physics HL Paper 3 Markscheme

Candidates are required to answer **all** questions in Section A and **all** questions from **one** option in Section B. Maximum total = **45 marks**.

1. Each row in the “Question” column relates to the smallest subpart of the question.
2. The maximum mark for each question subpart is indicated in the “Total” column.
3. Each marking point in the “Answers” column is shown by means of a tick (✓) at the end of the marking point.
4. A question subpart may have more marking points than the total allows. This will be indicated by “**max**” written after the mark in the “Total” column. The related rubric, if necessary, will be outlined in the “Notes” column.
5. An alternative wording is indicated in the “Answers” column by a slash (/). Either wording can be accepted.
6. An alternative answer is indicated in the “Answers” column by “**OR**”. Either answer can be accepted.
7. An alternative markscheme is indicated in the “Answers” column under heading **ALTERNATIVE 1 etc.** Either alternative can be accepted.
8. Words inside chevrons « » in the “Answers” column are not necessary to gain the mark.
9. Words that are underlined are essential for the mark.
10. The order of marking points does not have to be as in the “Answers” column, unless stated otherwise in the “Notes” column.
11. If the candidate’s answer has the same “meaning” or can be clearly interpreted as being of equivalent significance, detail and validity as that in the “Answers” column then award the mark. Where this point is considered to be particularly relevant in a question it is emphasized by **OWTTE** (or words to that effect) in the “Notes” column.
12. Remember that many candidates are writing in a second language. Effective communication is more important than grammatical accuracy.
13. Occasionally, a part of a question may require an answer that is required for subsequent marking points. If an error is made in the first marking point then it should be penalized. However, if the incorrect answer is used correctly in subsequent marking points then **follow through** marks should be awarded. When marking, indicate this by adding **ECF** (error carried forward) on the script. “ECF acceptable” will be displayed in the “Notes” column.
14. Do **not** penalize candidates for errors in units or significant figures, **unless** it is specifically referred to in the “Notes” column.

## Section A

Question		Answers		Notes	Total
1.	a		$[\mu] = \frac{\text{kg} \times \text{m s}^{-2}}{\text{s}^{-2} \times \text{m}^2} \Rightarrow \text{kg m}^{-1}$ ✓	Accept kg/m . Do <b>not</b> accept g m <sup>-1</sup> .	1
1.	b	i	straight line through all error bars ✓		1
1.	b	ii	line does not go through the origin «and all error bars» ✓		1
1.	b	iii	mass of tray of weights neglected/friction at pulley/friction at slider/thickness of slider/zero off-set error ✓	Do <b>not</b> allow vague answers like friction neglected / error in length measurement.	1
1.	b	iv	large enough triangle $\Delta m \geq 50 \text{ g}$ ✓  answer in range $0.210 - 0.240 \text{ kg m}^{-2}$ ✓	Accept answers in g m <sup>-2</sup> .  Do <b>not</b> allow ECF from MP1.	2
1.	c		$\mu = \frac{g \times \text{gradient}}{f^2} = 6.229 \times 10^{-4} \text{ kg m}^{-1}$ ✓  percentage uncertainty = $6.0 + 2 \times 2.0 = 10\%$ ✓	Accept $\mu$ in the range 5.7 to 6.6. Allow answer in g m <sup>-1</sup> . Allow ECF from (b).	2

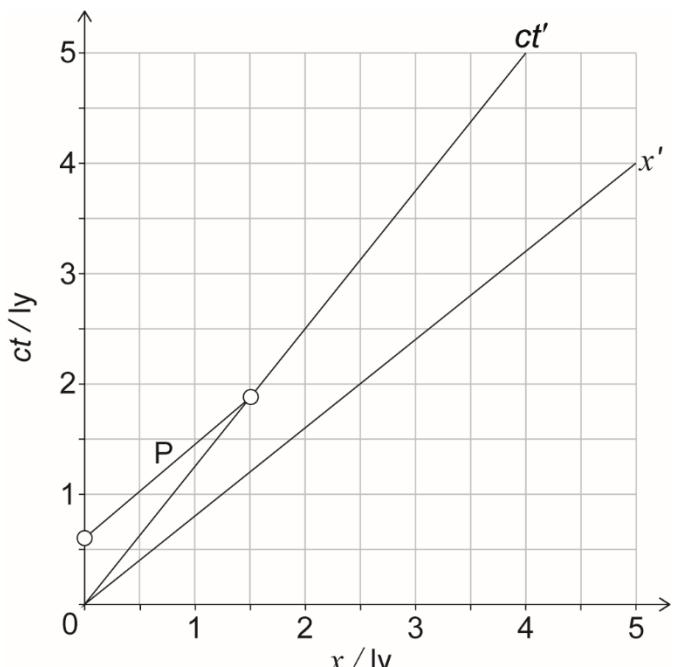
Question			Answers	Notes	Total
2.	a	i	mass <b>OR</b> diameter <b>OR</b> material of bob <b>OR</b> « initial » amplitude/angle ✓	<i>Do not allow statements about rulers, stopwatches, string, number of oscillations, constant gravity.</i>	1
2.	a	ii	student's reaction time «in starting and stopping stopwatch» / starting/stopping stopwatch ✓		1
2.	b		it reduces «the random» error/uncertainty ✓ by a factor of 20 «compared to that in a single period measurement» ✓	<i>For MP1, allow increasing accuracy/precision.</i>  <i>Award [0] for answers related to number of trials, 20 measurements of one period.</i>	2
2.	c	i	$T^2$ ✓		1
2.	c	ii	$g = 9.7 \text{ ms}^{-2}$ ✓ $\Delta g = 0.8 \text{ ms}^{-2}$ ✓		2

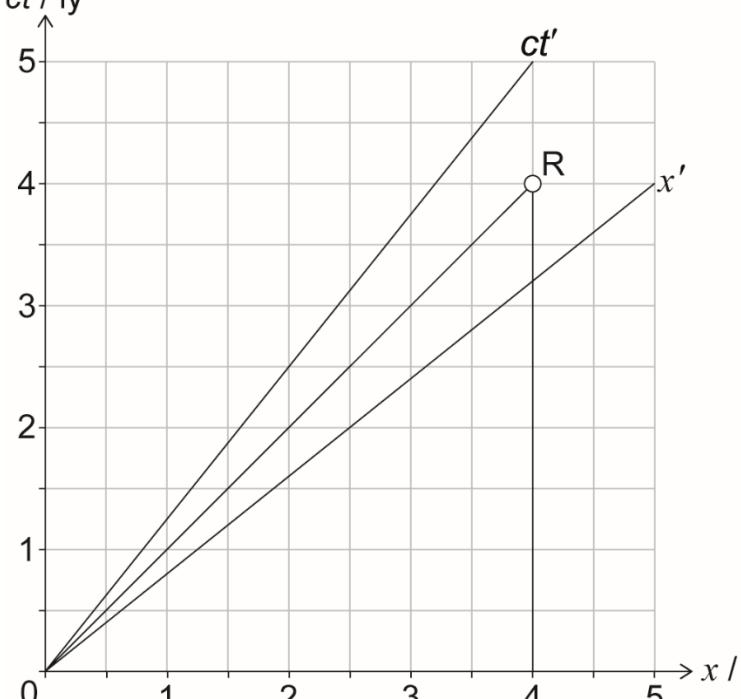
## Section B

### Option A — Relativity

Question			Answers	Notes	Total
3.	a		«in the proton rest frame » the speed of the proton is zero so force is zero ✓		1
3.	b		the proton sees an electric field / experiences electric force✓ upward force✓		2

Question			Answers	Notes	Total
4.	a		moves 4 ly in 5 years <b>OR</b> slope of angle with time axis is 0.8✓	Allow evidence for mark on the graph.	1
4.	b	i	$\gamma = 1.67$ <b>OR</b> $\frac{5}{3}$ <b>OR</b> $\frac{1}{\sqrt{(1 - 0.8^2)}}$ ✓ $ct' = \gamma(ct - \frac{vx}{c}) = \frac{5}{3} \times (0.60 + 0)$ ✓ «=1.00 ly»	For <b>MP2</b> , working should be seen.	2

Question		Answers	Notes	Total
4.	b ii	<p>identifies point with coordinates <math>x=0</math>, <math>ct=0.60</math> on vertical axis ✓</p> <p>draws line parallel to the <math>x'</math> prime axis until it intersects the prime <math>ct</math> axis ✓</p> 	<p><b>Award [2]</b> for correct position of <math>P</math> without working shown.</p>	2

Question			Answers	Notes	Total
4.	c	i	<p><math>ct / \text{ly}</math></p>  <p>R located at (4,4) ✓</p> <p>«as intersection of» vertical line through 4 ly and photon worldline at 45 degrees✓</p>	<p>Allow <b>MP2</b> even if one of the lines is not drawn.</p>	2

Question			Answers	Notes	Total
4.	c	ii	<p><b>ALTERNATIVE 1</b></p> <p><i>Using diagram:</i></p> <p>line from R parallel to prime ct axis until it intersects space axis ✓</p> <p>use of scale from (b) to estimate coordinate to <math>x' = (1.3 \pm 0.2)</math> ly ✓</p> <p><b>ALTERNATIVE 2</b></p> <p><i>Using Lorentz transformation:</i></p> <p>event R has coordinates <math>x = ct = 4.00</math> ly in S ✓</p> <p>so <math>x' = \gamma(x - vt) = \frac{5}{3} \times (4.00 - 0.80 \times 4.00) = 1.33</math> ly ✓</p>		2

Question			Answers	Notes	Total
5.	a		$T_{\frac{1}{2}} = 2.00 \times 1.56 \times 10^{-6}$ or $3.12 \times 10^{-6}$ s ✓ $D = «3.12 \times 10^{-6} \times 0.866 \times 3 \times 10^8» = »811 \text{ m}»$ ✓	Award [2] for BCA	2
5.	b	i	$0.866c \times \text{half life} = 0.866c \times 1.56 \times 10^{-6} = «405 \text{ m}»$ <b>OR</b> distance travelled by detector $= \frac{D}{y} = \frac{D}{2} = «405 \text{ m}»$ ✓	Working must be seen.	1
5.	b	ii	transit time $= \frac{405}{0.866c} = 1.56 \mu\text{s}$ ✓	Award [1] for BCA.	1
5.	c		transit time is one half life ✓ so ratio has to be $\frac{1}{2}$ ✓	Award [2] for BCA.	2
5.	d		the answers are the same ✓ count rates cannot vary from frame to frame /OWTTE ✓	Do not allow ECF from (c).  Award [2] for “count rates cannot vary” if student made a mistake OR no answer in (c) and well discussed here.	2

Question		Answers	Notes	Total
6.	a	$E_T = «0.938 + 3.40 =» 4.34 \text{ GeV} \checkmark$ attempted use of $p^2 c^2 = E^2 - m_0^2 c^4$ « $pc = \sqrt{4.34^2 - 0.938^2}$ » $\checkmark$ $p = 4.24 \text{ GeV c}^{-1} \checkmark$	<b>Award [2] if omitted rest mass in MP1, the answer is 3.27.</b>	3
6.	b	$\gamma = «\frac{4.34}{0.938} =» 4.63 \checkmark$ $v = 0.976c \checkmark$	Allow ECF from (a).	2

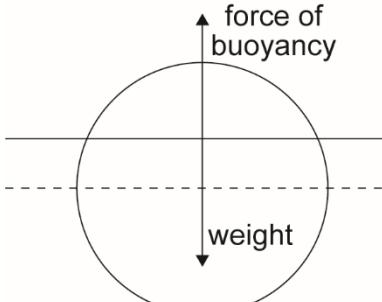
Question		Answers	Notes	Total
7.	a	<p>according to the EP the tower is equivalent to a frame accelerating away from Earth «with <math>a = g</math>» ✓</p> <p>an observer at B approaches the source of light ✓</p> <p>and so by the Doppler effect must measure a higher frequency ✓</p>	<i>Award [1] for correct explanation without principle of equivalence,</i>	3
7.	b	<p>«the clock at B» runs slower «than T» ✓</p> <p>the period at B is shorter since frequency is higher <b>OR</b> Clock B is in a stronger gravitational field than T <b>OR</b> the wave red shift when travelling up ✓</p>		2

## Option B — Engineering physics

Question			Answers	Notes	Total
8.	a		$0.180 + 0.200 \times 0.60^2 \ll= 0.252 \text{ kg m}^2$ ✓		1
8.	b		angular speed of particle = « $12/0.6 = » 20 \text{ rad s}^{-1}$ » <b>OR</b> angular momentum of particle « $0.200 \times 12.0 \times 0.60 » = 1.44 \text{ Js}$ » ✓  «angular momentum of rod-particle system 0.252 $\omega$ » equating $\omega = \frac{1.44}{0.252} » = 5.71 \text{ rad s}^{-1}$ ✓	<i>For MP2, working or answer to at least 3 SF should be seen.</i>	2
8.	c		$\frac{1}{2} \times 0.200 \times 12.0^2 - \frac{1}{2} (0.252) \times 5.71^2$ ✓ 10.3 J ✓	<b>Award [1]</b> for answer 11.5 J that neglects moment of inertia of particle but do not penalize this omission in (d)(i).	2
8.	d	i	$\alpha = \frac{0.152}{0.252} = 0.603 \text{ rad s}^{-2}$ ✓	Accept negative values.	1
8.	d	ii	$\theta = \frac{5.71^2}{2 \times 0.603} = 27.0 \text{ rad}$ ✓ $N = \frac{27.0}{2 \times \pi} = 4.3$ ✓		2
8.	e		the rod will rotate «about centre of mass» ✓  «centre of mass» will move along straight line «parallel to the particle's initial velocity» ✓	<i>For MP2, mention of translational motion is not enough.</i>	2

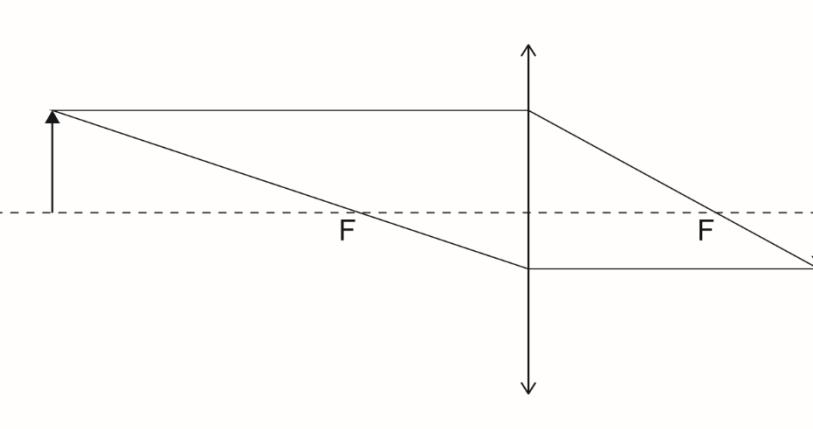
Question		Answers	Notes	Total
9.	a	<p><b>ALTERNATIVE 1</b>          «considering expansions from A» an adiabatic process will reduce/change temperature ✓</p> <p>and so curve AC must be the steeper ✓</p> <p><b>ALTERNATIVE 2</b>          temperature drop occurs for BC ✓          therefore CA must increase temperature «via adiabatic process». ✓</p>		2
9.	b	<p><b>ALTERNATIVE 1</b></p> <p>Use of adiabatic formula <math>p_A V_A^{\frac{5}{3}} = p_C V_C^{\frac{5}{3}} \Rightarrow V_C = \left(\frac{p_A}{p_C}\right)^{\frac{3}{5}} V_A</math> ✓</p> $V_C = \left(\frac{5.00 \times 10^5}{4.60 \times 10^3}\right)^{\frac{3}{5}} \times 2.00 \times 10^{-3} \approx 3.333 \times 10^{-2} \text{ m}^3$ ✓ <p><b>ALTERNATIVE 2</b></p> <p><math>V_C = V_B</math> AND <math>p_A V_A = p_B V_B</math> ✓</p> $V_C = \frac{5 \times 10^5 \times 2 \times 10^{-3}}{3 \times 10^4}$ ✓ <p><b>ALTERNATIVE 3</b></p> <p><math>V_C = V_B</math> AND <math>n=0.2 \text{ mol}</math> ✓</p> $V_C = (0.2 \times 8.31 \times 602) / 4 \times 10^4$ ✓	<i>For MP2, working or answer to at least 4 SF must be seen.</i>	2

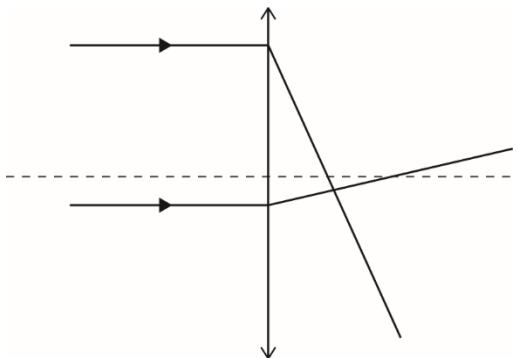
Question		Answers	Notes	Total
9.	c	<p>Increasing ✓</p> <p>because thermal energy/heat is being provided to the gas « and temperature is constant, <math>\Delta S = \frac{\Delta Q}{T}</math> » ✓</p>		2
9.	d	<p><b>ALTERNATIVE 1</b></p> $Q = \Delta U = \frac{3}{2} V_c \Delta P \checkmark$ $Q = \left\langle \frac{3}{2} \times 3.33 \times 10^{-2} \times (3.00 \times 10^4 - 4.60 \times 10^3) \right\rangle = 1268.7 \approx 1270 \text{ «J» } \checkmark$ <p><b>ALTERNATIVE 2</b></p> $Rn = \frac{5 \times 10^5 \times 2 \times 10^{-3}}{602} = 1.66 \quad \text{OR} \quad T_c = 4.6 \times 10^3 \times 3.33 \times 10^{-2} \times 1.66 = 92.2 \checkmark$ $\Delta U = \frac{3}{2} \times 1.661 \times (602 - 92.21) = 1270 \text{ «J» } \checkmark$	<p><b>Award [2] for BCA.</b> Accept negative values.</p> <p><b>Award MP1 if <math>T_c = 92</math> taken from (e)</b></p>	2
9.	e	$e_c = 1 - \frac{92}{602} = 0.847 \checkmark$ <p>this engine has <math>e &lt; e_c</math> as it should ✓</p>	<p><b>Award [0] if no calculation shown.</b></p>	2

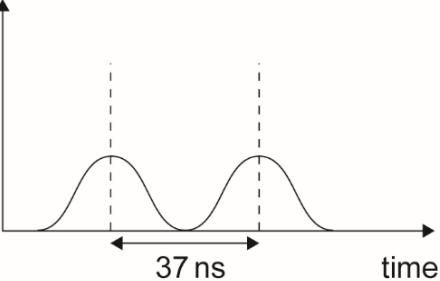
Question			Answers	Notes	Total
10.	a		 <p>buoyancy force greater than weight ✓</p>	<p><b>Award [1]</b> for correct labeling <b>AND</b> relative size. Allow any point, where the forces are drawn.</p>	1
10.	b	i	<p>considering consecutive amplitudes ✓</p> $Q = \left\langle 2\pi \frac{2.0^2}{2.0^2 - 1.5^2} \right\rangle = 14 \checkmark$		2
10.	b	ii	<p>Q will decrease ✓</p> <p>higher viscosity provides more damping/higher energy dissipated per cycle ✓</p>		2

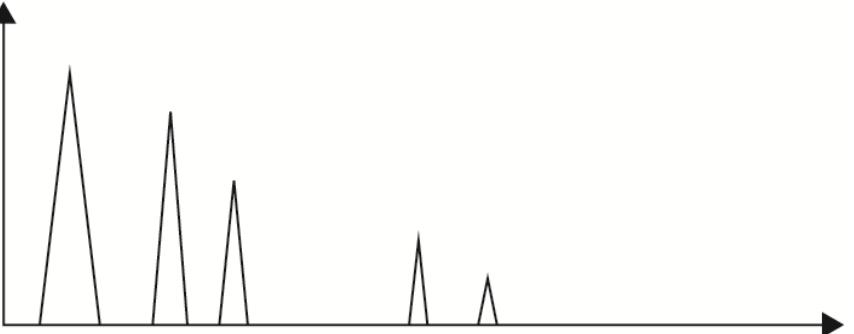
Question		Answers	Notes	Total
11.	a	<p>«considering a streamline joining the surface of the oil to B»</p> <p>«<math>0 + 0 + P_{atm} = -\rho g H + P_{atm} + \frac{1}{2} \rho v^2</math> ✓</p> <p><math>v = \sqrt{2g \times H} = \sqrt{2 \times 9.81 \times 8.0}</math> ✓</p> <p>«= 12.53 m s<sup>-1</sup>»</p>	<p>Award <b>1 max</b> for use of Torricelli theorem.</p> <p>Do <b>not</b> accept a BCA, MP1 must be seen.</p>	2
11.	b	«by the equation of continuity $v = \text{const}$ » because the diameter/area is constant ✓		1
11.	c	<p>setting pressure at highest point to zero gives «<math>0 + 0 + P_{atm} = \rho gh + 0 + \frac{1}{2} \rho v^2</math> ✓</p> <p><math>h = \left( \frac{P_{atm} - \frac{1}{2} \rho v^2}{\rho g} \right) = \frac{1.01 \times 10^5 - \frac{1}{2} \times 915 \times 12.5^2}{915 \times 9.81}</math> » = 3.29 m ✓</p>		2

## Option C — Imaging

Question		Answers	Notes	Total
12.	a	 <p>one of the two rays above ✓</p>		1
12.	b	$-\frac{v}{u} = -\frac{1}{2} \text{ OR } v = \frac{u}{2} \checkmark$ $\frac{1}{u} + \frac{2}{u} = \frac{1}{4.0} \checkmark$ $u=12\text{cm} \checkmark$	<p>Diagram is <b>not</b> to scale so award [0] if answer obtained by measurement.</p> <p>Allow <b>MP1</b> if mistake in negative sign.</p> <p>Do <b>not</b> allow <b>ECF</b> from <b>MP1</b>.</p> <p>Award [3] for <b>BCA</b>.</p>	3

Question			Answers	Notes	Total
12.	c	i	 <p>the extreme ray crosses principal axis closer than paraxial ray ✓</p>		1
12.	c	ii	image is curved / blurred / distorted / poorly focused ✓		1
12.	c	iii	block non-paraxial rays/ reduce aperture/use rays closer to axis/ <b>OWTTE</b> <b>OR</b> use aspherical lens ✓	<i>Allow parabolic lens.</i> <i>Allow use of additional lens OR compensation plates.</i>	1
12.	d	i	angular magnification is $\frac{75}{4.0} = 18.75$ ✓ angle = « $18.75 \times 0.51^\circ$ » = $9.6^\circ$ ✓		2
12.	d	ii	It would be «much» smaller ✓		1

Question			Answers	Notes	Total
13.	a	i	$1.500 \times \sin \theta_c = 1.489 \times 1 \checkmark$ $\theta_c = 83.06^\circ \checkmark$	Allow $83^\circ$ OR $1.45 \text{ rad.}$	2
13.	a	ii	$1.00 \times \sin \theta_{\max} = 1.500 \times \sin(90^\circ - 83.06^\circ) \checkmark$ $\theta_{\max} = 10.4^\circ \checkmark$	Allow $11^\circ$ Allow ECF from (a)(i).	2
13.	b		« when ray 1 travels a distance 1.00 km » ray 2 travels a distance $\frac{1}{\sin \theta_c} = 1.0074 \text{ km} \checkmark$ speed is $\frac{c}{1.500} = 2.0 \times 10^8 \text{ ms}^{-1} \checkmark$ hence $\Delta t = \frac{10^3}{2 \times 10^8} = 36.67 \text{ ns} \checkmark$	Allow ECF from (a)(i). Time for ray 1 is 5000 ns , time for ray 2 is 5036.9 ns. Do not allow ECF from MP2. For MP3, working or answer to at least 3 SF should be seen.	3
13.	c	i	amplitude  pulses rounded and of lower amplitude, symmetrical about dotted lines. just starting to overlap $\checkmark$		2
13.	c	ii	the pulses would overlap/the signal will be lost/OWTTE $\checkmark$		1

Question			Answers	Notes	Total
14.	a		«pulses of» ultrasound are reflected « from organ boundaries» ✓ «depth information is obtained from elapsed» times between transmission and reception of reflected waves ✓		2
14.	b	i	 4 reflected pulses ✓ separated in time and shorter than incident pulse ✓	<i>Allow 3 reflected pulses because there may be little reflection from the front of the cornea due to the gel.</i>	2
14.	b	ii	by measuring «the half» time «between the 2 pulses to the right» ✓ and multiplying by the speed of sound «in eye lens» ✓		2

Question			Answers	Notes	Total
14.	c	i	<p>the RF signal is at the resonant frequency or Larmor frequency of the «precessing» protons ✓</p> <p>the protons will «absorb the energy and» make a transition to the higher energy / spin down» state ✓</p>		2
14.	c	ii	<p><b>ALTERNATIVE 1</b></p> <p>the time taken for the nuclei to return to their original states/intensity of signal is measured ✓</p> <p>these are different for different tissues✓</p> <p><b>ALTERNATIVE 2</b></p> <p>use of the gradient field «allows location of emitted signal» ✓</p> <p>measured intensity is dependent on different tissues ✓</p>		2

**Option D — Astrophysics**

Question		Answers	Notes	Total
15.		<p>open clusters are irregular in shape <b>OR</b> globular clusters are spherical ✓</p> <p>open clusters have few stars <b>OR</b> globular clusters have many ✓</p> <p>open clusters have young stars <b>OR</b> globular clusters have old stars ✓</p> <p>open clusters have active stars <b>OR</b> globular clusters have mostly dead stars ✓</p> <p>open clusters are less dense <b>OR</b> globular clusters are more dense ✓</p>	<p><i>Open clusters are bigger / have stars further apart is not enough to award the mark.</i></p> <p><i>Globular clusters are round is not enough to award the mark.</i></p>	1 max

Question			Answers	Notes	Total
16.	a		$L = L_{\odot} \times 52^{3.5} = 1.01 \times 10^6 L_{\odot} \checkmark$	Accept back working. Working should be seen.	1
16.	b		$L_B = L_A = 10^6 L_{\text{sun}} \checkmark$ $\frac{L_B}{L_{\odot}} = \left(\frac{4\pi R_B^2}{4\pi R_{\odot}^2}\right) \left(\frac{3.0}{6.0}\right)^4 \quad \text{OR} \quad 1.0 \times 10^6 = \left(\frac{R_B}{R_{\odot}}\right)^2 \frac{1}{16} \checkmark$ $\frac{R_B}{R_{\odot}} = \sqrt{1.0 \times 10^6 \times 16} = 4 \times 10^3 \checkmark$	Award [3] for BCA  Do not allow ECF from MP2 to MP3.	3
16.	c		«star A will evolve into a» red supergiant star $\checkmark$ will then explode/supernova $\checkmark$ creating a neutron star or black hole $\checkmark$ « neutron star if core/remnant mass < the Oppenheimer–Volkoff limit/ a black hole if core more massive »	For MP3, award the mark if only one of neutron star OR black hole is mentioned.	3
16.	d		$d = \sqrt{\frac{L}{4\pi b}} \checkmark$ $d = \sqrt{\frac{2.4 \times 10^{23}}{4\pi \times 4.1 \times 10^{-14}}} \ll = \frac{6.8 \times 10^{17}}{3.09 \times 10^{16}} \gg \approx 22 \text{ pc} \checkmark$	Award [2] for BCA.  For MP1, allow answer in unit m, value $6.8 \times 10^{17} \text{ m}$ .	2
16.	e		the distance is well within the limit of about 1000 pc for stellar parallax, so yes it can $\checkmark$	Accept 100 pc OR similar distance in other units.  Allow ECF from (d) (also converse argument).	1

Question			Answers	Notes	Total
17.	a		$\frac{v}{c} = \frac{512 - 486}{486} \checkmark$ $\Rightarrow v = 1.6 \times 10^4 \text{ «km s}^{-1}\text{»} \quad \checkmark$	Accept 0.053 c. Accept answer in m s <sup>-1</sup> . <b>Award [2] for BCA.</b>	2
17.	b		$d = \frac{v}{H} = \frac{1.6 \times 10^4}{72} = 222 \approx 220 \text{ «Mpc»} \checkmark$	<b>ECF from (a).</b>	1
17.	c		it is important because it is related to the age of the universe ✓ it is important in testing models of the universe ✓ it is important for determining distances ✓		1 max

18.	a		$\text{«}z = \frac{R}{R_0} - 1 \Rightarrow \text{»} \frac{R_0}{R} = \frac{1}{1+z} = \frac{1}{2.74} = 0.36496 \checkmark$		1
18.	b	i	«physical lengths are proportional to R so» $\frac{\lambda}{R} = \frac{\lambda_0}{R_0} \checkmark$ use of Wien's law $\lambda T = \text{const}$ to get result✓		2
18.	b	ii	$T = T_0 \frac{R_0}{R} \text{ OR } \frac{T}{T_0} = \frac{R_0}{R} \text{ OR } TR = T_0 R_0 \checkmark$ $T = 7.4 \times 0.365 = 2.7 \text{ «K»} \checkmark$		2

Question			Answers	Notes	Total
19.	a		$V = \frac{4\pi R^3}{3} \ll= \frac{4\pi (9 \times 10^{15})^3}{3} = 3.05 \times 10^{48} \text{ m}^3 \checkmark$ $M = 3.05 \times 10^{48} \times 10^{10} \times 1.67 \times 10^{-27} = 5.1 \times 10^{31} \text{ kg} \checkmark$	For MP2, values must be substituted to show working.	2
19.	b		$M_J = «1.8 \times 10^{33} \left(\frac{50}{100}\right)^{1.5} » = »6.4 \times 10^{32} \text{ kg} \checkmark$ <p>according to the Jeans criterion collapse will start when the mass of the star is greater than the Jeans mass <math>\checkmark</math></p> <p>«which is not the case here»</p>		2
19.	c	i	$1.8 \times 10^{33} \left(\frac{T}{100}\right)^{1.5} = 5.1 \times 10^{31} \checkmark$ $T \approx 9 \text{ K} \checkmark$		2
19.	c	ii	the temperature at the core must be high enough for fusion «of hydrogen» to take place $\checkmark$		1
19.	d		$M^{3.5} \propto \frac{M}{t} \checkmark$ $\frac{t}{t_{\square}} = \left(\frac{M_{\square}}{M}\right)^{2.5} \checkmark$ $t = \left(\frac{2 \times 10^{30}}{5 \times 10^{31}}\right)^{2.5} \times 8 \times 10^9 = 2.6 \times 10^6 \text{ years} \checkmark$	Award [3] for correct alternative solution OR BCA from interval 2.2 to 2.8.	3